NOTATION

ζ, hydrodynamic drag coefficient; p, v, T, ρ, pressure, specific volume, temperature, and density; R, universal gas constant; μ, molecular mass; C_{pef}, effective isobaric mass specific heat; k_T, adiabatic temperature index of dissociating gas; p_0, T_0, ρ_0, stagnation parameters; α, degree of dissociation; k, adiabatic index of nondissociating ideal gas; w, velocity of dissociating nitrogen tetroxide; G, flow rate per second; F, cross section; a_{cr}, critical velocity of dissociating nitrogen tetroxide; λ, velocity coefficient.

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DYNAMIC VISCOSITY OF GASES OVER A WIDE RANGE OF TEMPERATURES. I.

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UDC 533.16

The results are presented of a generalization of the experimental data on the viscosities of He, Ne, Ar, Kr, Xe, and N_2 at atmospheric pressure over the range of temperatures from the boiling point at atmospheric pressure to 2000 K using polynomial approximating relationships.

The recommended value of the viscosities of the inert gases and nitrogen which exist at present [1-4] differ fundamentally in the methods of generalizing the experimental data, in the type and quantity of the block of data used in the generalization, in the form of representing the results, and in the magnitude of the errors inherent in them.

The results of the direct generalization of the experimental data on the viscosities of the gases [1] are characterized by errors not exceeding 1.5%, but they are based on an incomplete collection of data obtained only up to 1972.

The standard handbook data [2] are based on a reliable theoretical model, but the generalization of the experimental results was carried out only for argon, and for the other gases the data were obtained by using the principle of corresponding states. As shown in [5], this method can lead to inaccuracies for neon at temperatures above room temperature and for helium in the low temperature region. In fact, the deviation of the data of [2] from the experimental viscosity values is largest for Ne at $T > 300$ K compared to the other gases (Ar, Kr, Xe); as regards helium, the measured results for this gas were generalized graphically over the entire temperature range. The data of [2] on the viscosities and thermal conductivities had similar errors, though the errors of the initial experimental material were different.

The recalculation of thermal conductivity data into values of the viscosity carried out in [3] has the same deficiency: the error of the values which are obtained is caused by the

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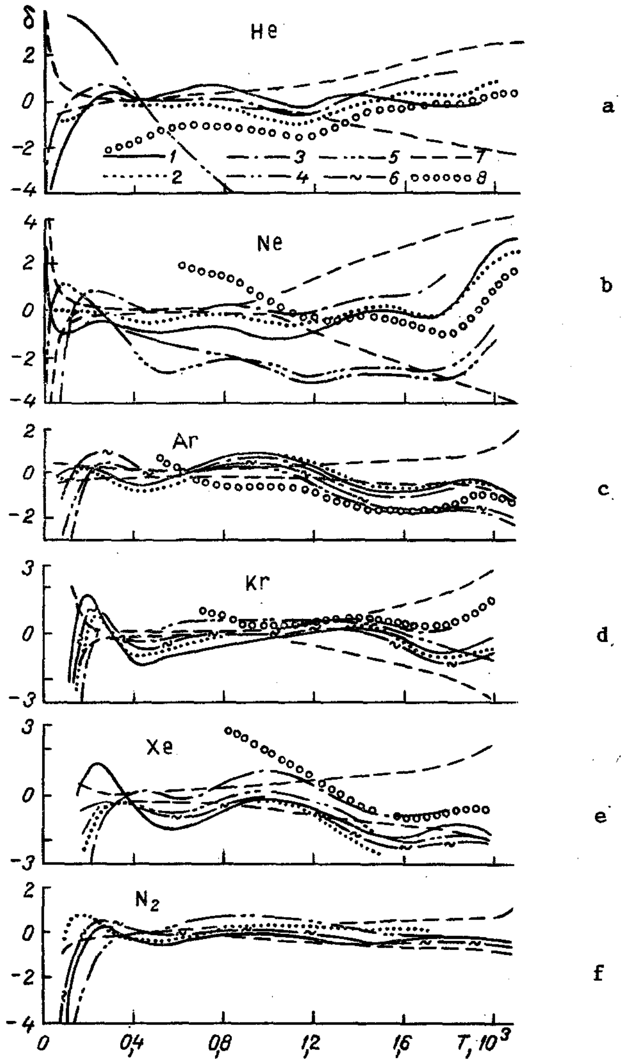

Fig. 1. Deviations between the recommended data and the theoretical values of the gas viscosities from the results of the generalization of the data of the following references: 1) [2, 4]; 2) [1]; 3) [11]; 8) [3]; 4-6) calculations using the (12-6), (12-7), and (m-6 – 8) potential functions respectively; 7) statistical corridor for the error of the generalized values with a confidence coefficient of 99%. $\delta = (\eta - \eta_{gen})/\eta_{gen}$, %; T is in K.

error of the thermal conductivity data, even though most of the viscosity measurements which are carried out at present have an accuracy an order of magnitude higher on average. The viscosity values in [3] at T = 2500-6000 K are the only high-temperature data so far, and in view of high errors are in need of confirmation and subsequent refining.

It follows from the discussion above that the existing recommended and handbook data [1-3] on the viscosities of the inert gases have definite inherent deficiencies, mainly in connection with the evaluation of the errors. The objective of the present paper is the determination of smoothed data on the viscosities of the inert gases and nitrogen on the basis of reliable experimental values (indicated in [6]) obtained recently, taking into account the authority of the experimental data, and to carry out a subsequent evaluation of the error of the results of the generalizations.

Polynomial regrssion was used for determining the temperature dependence of the coefficient of dynamic viscosity. The regression equation was set up in the form a polynomial of a single variable

$$y_i = a_0 + a_1 x_i + a_2 x_i^2 + \dots + a_p x_i^p \tag{1}$$

TABLE 1. Coefficients of the Polynomial in $\eta = \sum_{j=0}^{n} a_j \left(\frac{T}{1000}\right)^j$

Coefficient	He		Ne	Ar	Kr	Xe	N_2
	T, K					T, K	
	4,21—20	20—2200	27,09—2000	87,28—2100	119,78—2000	165,03—2100	77,35—2100
a_0	0,25486	1,39621	—0,31367	0,21317	6,58060	4,31630	—0,03652
a_1	255,72523	111,43828	193,86852	83,74835	—8,32792	33,01191	75,97879
a_2	—6365,53819	—407,88308	—625,47701	—15,97622	545,35888	182,72178	—67,13529
a_3	82228,08838	1501,91966	1946,58300	—64,37271	—1578,34402	—353,12700	48,15367
a_4		—3476,99966	—3933,98655	86,11881	2397,56055	300,02441	—18,15740
a_5		5057,02966	4992,97122	—41,46889	—2118,41131	—120,43154	2,72311
a_6		—4676,31566	—3927,33988	7,01619	1088,16236	18,51324	
a_7		2736,89755	1850,41822		—300,56796		
a_8		—979,76003	—477,07904		34,45380		
a_9		195,50742	51,65753				
a_{10}		—16,64656					

TABLE 2. Generalized Values of the Dynamic Viscosity (μPa·sec) of Helium at Atmospheric Pressure in the Very Low Temperature Zone

T, K	η	$\pm\Delta\eta$	T, K	η	$\pm\Delta\eta$
4,21	1,22	0,07	10	2,26	0,06
5	1,38	0,06	12	2,55	0,06
6	1,58	0,06	14	2,81	0,06
7	1,78	0,06	16	3,05	0,06
8	1,93	0,06	18	3,27	0,06
9	2,10	0,06	20	3,48	0,07

and the subsequent regression analyses were carried out using a standard program [7] on an ES-1060 computer. The computations were carried out using the method of orthogonal polynomials, which made it possible to surmount serious difficulties in the numerical calculations due to the poor conditioning of the regression matrix of Eq. (1) [8].

For selecting the polynomial use was made of a weighted least squares method consisting of minimizing the sum

$$S_p = \sum_{i=1}^{N} w_i (y_i - a_0 - a_1 x_i - \ldots - a_p x_i^p)^2,$$

where w_i is the weighting of the i-th point (i = 1, 2, ..., N). The value of the reciprocal of the square of the absolute error was selected as the weighting factor.

Starting with a polynomial of the first order and ending with a polynomial of order specified in advance, the program which was used made it possible to obtain estimates of the coefficients of the regression equation and their standard errors, the correlation matrix for the regression coefficients, the residual sums and the mean squares of the residuals, and the residual and value of the regression coefficient for the orthogonal polynomials [9]. The significance of the regression coefficients was tested by the t-criterion, and the degree of the polynomial was established by the F-criterion [9], as a result of which the equation was established with statistical values, i.e., with reliable coefficients.

The estimate of the total error Δy_i of the calculated value y_i included an analysis of the random, systematic, and computational errors.

The contribution of the random errors $\sigma_{i\mathrm{ran}}$ in the predicted value of y_i computed from the regression with statistical values of the coefficients was calculated using the covariance matrix G of the coefficients

$$\sigma_{i\mathrm{ran}} = (r^T G r)^{1/2},$$

where $r = (1, x_i, x_i^2, \ldots, x_i^{n-1})$.

TABLE 3. Generalized Values of the Dynamic Viscosities (μPa·sec) of Gases at Atmospheric Pressure

T, K	η	±Δη	η	±Δη	η	±Δη	η	±Δη	η	±Δη	η	±Δη
	He		Ne		Ar		Kr		Xe		N_2	
20	3,47	0,10										
27,09			4,51	0,39								
30	4,41	0,08	4,99	0,38								
40	5,29	0,07	6,56	0,32								
50	6,12	0,05	8,04	0,24								
60	6,90	0,05	9,44	0,17								
70	7,64	0,05	10,77	0,12								
77,35											5,46	0,04
80	8,34	0,04	12,04	0,09							5,64	0,04
87,28					7,36	0,03						
90	9,02	0,04	13,26	0,07	7,58	0,03					6,29	0,04
100	9,66	0,04	14,42	0,06	8,37	0,03					6,94	0,03
110	10,28	0,04	15,53	0,06	9,16	0,03					7,57	0,03
119,78							11,14	0,27				
120	10,88	0,03	16,60	0,05	9,94	0,03	11,15	0,27			8,19	0,03
130	11,46	0,03	17,64	0,05	10,71	0,03	11,85	0,22			8,81	0,02
140	12,03	0,03	18,64	0,05	11,48	0,03	12,59	0,16			9,41	0,02
150	12,58	0,03	19,61	0,04	12,24	0,03	13,34	0,13			10,00	0,02
160	13,12	0,03	20,55	0,04	12,99	0,03	14,11	0,10			10,59	0,02
165,03									13,36	0,09		
170	13,64	0,03	21,47	0,04	13,74	0,03	14,90	0,09	13,71	0,08	11,16	0,02
180	14,16	0,03	22,36	0,04	14,48	0,03	15,70	0,08	14,41	0,07	11,73	0,02
190	14,67	0,03	23,23	0,03	15,21	0,03	16,51	0,07	15,12	0,06	12,28	0,02
200	15,17	0,02	24,09	0,03	15,93	0,03	17,33	0,06	15,85	0,05	12,83	0,02
220	16,16	0,02	25,75	0,03	17,36	0,03	18,98	0,05	17,31	0,04	13,90	0,02
240	17,12	0,02	27,35	0,03	18,76	0,03	20,64	0,04	18,79	0,04	14,94	0,02
260	18,07	0,01	28,90	0,02	20,12	0,02	22,29	0,03	20,28	0,03	15,95	0,01
280	19,00	0,01	30,42	0,02	21,46	0,02	23,93	0,02	21,78	0,03	16,92	0,01
300	19,92	0,01	31,89	0,02	22,76	0,02	25,55	0,01	23,28	0,03	17,87	0,01
400	24,34	0,02	38,80	0,03	28,85	0,03	33,16	0,03	30,68	0,05	22,26	0,02
500	28,46	0,03	45,06	0,05	34,24	0,04	39,90	0,05	37,64	0,09	26,14	0,03
600	32,30	0,04	50,78	0,06	39,07	0,05	45,92	0,06	44,01	0,13	29,64	0,04
700	35,98	0,05	56,13	0,08	43,46	0,06	51,47	0,07	49,81	0,18	32,86	0,05
800	39,57	0,07	61,29	0,12	47,55	0,08	56,72	0,09	55,15	0,24	35,88	0,07
900	43,12	0,12	66,35	0,16	51,46	0,10	61,72	0,11	60,17	0,30	38,76	0,09
1000	46,58	0,19	71,30	0,30	55,28	0,12	66,47	0,15	65,03	0,39	41,52	0,13
1100	49,89	0,29	76,06	0,53	59,06	0,16	70,95	0,23	69,82	0,47	44,19	0,16
1200	53,00	0,40	80,53	0,91	62,81	0,21	75,21	0,37	74,59	0,57	46,79	0,19
1300	55,91	0,51	84,70	1,29	66,52	0,27	79,33	0,57	79,32	0,67	49,32	0,22
1400	58,70	0,65	88,66	1,66	70,14	0,33	83,42	0,80	83,95	0,81	51,77	0,26
1500	61,47	0,79	92,60	2,02	73,62	0,38	87,58	1,01	88,34	0,94	54,15	0,30
1600	64,28	0,95	96,63	2,43	76,91	0,43	91,78	1,24	92,52	1,13	56,46	0,34
1700	67,12	1,13	100,65	2,84	79,98	0,50	95,90	1,50	96,33	1,35	58,70	0,38
1800	69,90	1,35	104,32	3,39	82,87	0,58	99,70	1,91	99,89	1,63	60,88	0,43
1900	72,50	1,55	107,20	4,20	85,71	0,65	103,05	2,29	103,48	1,99	63,04	0,50
2000	74,89	1,76	109,67	4,95	88,76	0,73	106,28	3,16	107,64	2,54	65,23	0,59

The systematic error σ_{isys} was evaluated using a mathematical experiment [10]. For this purpose the initial experimental data were repeatedly artificially distorted in accordance with their possible systematic errors. The final coefficients of the regression equation were obtained by averaging over all the series of the mathematical experiment which were carried out:

$$a = \frac{\sum_{l=0}^{L} S_l^2 a_l}{\sum_{l=0}^{L} S_l^2},$$

where S_ℓ^2 is the residual variance of the ℓ-th variant of the distortion of the mass of experimental material.

The evaluation of the systematic error $\sigma_{isys} = (r^TQr)^{1/2}$ was carried out using the covariance matrix

$$Q = \frac{1}{L} \sum_{l=0}^{L} (S_l a_l - a)(S_k^2 a_k - a),$$

where k = 0, 1, 2, ..., L.

In evaluating the total error Δy_i the computational error connected with the errors in realizing the computational procedures of the least squares method were neglected because of their small values.

Thus, the error of the generalized value y_i of the viscosity was obtained by summing the errors

$$\Delta y_i = \pm t(\sigma_{\text{iran}} + \sigma_{\text{isys}}),$$

where the value of t was determined from tables of Student's distribution. With a 99% confidence coefficient, t = 3 for the case being considered where there is a large number of experimental points (232 values for He, 171 for Ne, 307 for Ar, 144 for Kr, 94 for Xe, and 302 for N_2).

The numerical values of the coefficients a_j of the polynomials $\eta = \sum_{j=0}^{n} a_j \left(\frac{T}{1000}\right)^j$, μPa·sec, which approximate the temperature dependences of the viscosities of the inert gases and nitrogen according to the experimental data indicated in [6] are shown in Table 1. Tables 2 and 3 give the generalized values of the viscosity which are obtained and the estimated errors for these ranges at a confidence coefficient of 99%.

The deviations of the experimental data on viscosity used during smoothing from the generalized values which are obtained which were found in [6] are shown in Figs. 1a-f, from which it can be seen that the level of agreement of the results varies with the temperature range: as a rule, the deviation is greatest at the ends of the temperature range investigated, where the experimental data are few in number and suffer form systematic errors.

The comparison of the viscosity data shown in Fig. 1 indicates that the results of our generalization are in agreement with the standard handbook data [2, 4] in practice within the range of their total errors over the entire range which has been considered except for the low temperature zone. At the low temperatures the data of [2, 4] for He, Ar, Kr, and N_2 fall below and those for Ne fall above out generalized values. It can also be seen from Fig. 1 that the results of the generalization of the experimental values of the viscosity are overestimated relative to the theoretical values at the low temperatures, which in our opinion is caused by the effect of the non-ideality of the gases on the process of momentum transfer; in order to overcome this deviation in the theoretical values it is necessary to take into account a correction to the density, as was done in [12], for example, in order to produce agreement of the theoretical and experimental data on the thermal conductivities of the heavy inert gases at low temperatures using a quasichemical procedure. This same reason is the explanation for the deviation observed at low temperatues in the standard handbook data [4] for rarefied nitrogen and also for the results [2, 13] for He, for which at temperatures below 30 K we had to depend mainly on experimental values obtained at pressures close to atmospheric pressure.

The following can be mentioned in support of the smoothed values which have been obtained for the viscosities of the inert gases and nitrogen: the completeness and reliability of the initial block of experimental data; the method of taking into account the errors of the experimental data in the form of weightings during their treatment; and the subsequent evaluation of the errors in the results of the generalization.

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GENERALIZATION OF EXPERIMENTAL DATA ON THERMAL CONDUCTIVITY OF PARAFFIN SERIES HYDROCARBONS AND ALCOHOLS

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UDC 536.2

An expression is obtained for calculating the temperature dependence of the thermal conductivity coefficient of vapors of the paraffin series hydrocarbons and their alcohols over a wide temperature range at pressures of P = 0.1 MPa.

The present state of precise kinetic theory [1] does not provide results accurate enough for practical applications in calculating the thermal conductivity coefficient of multiatomic gases. The theory has been developed only for spherically symmetric molecules without internal degrees of freedom, i.e., monatomic gas molecules.

The absence of a reliable analytical method for calculating λ of multiatomic gases makes experiment the major source of data on thermal conductivity of these substances. However the expensive and inefficient process of purely empirical study of thermal properties of substances with complex molecular structure cannot be extended without limit. It thus becomes necessary to produce computation expressions which permit determination of the transport properties of materials over a wide parameter range with an accuracy close to that of experiment. In the author's opinion the most efficient method of solving this problem is the similarity theory-based method of generalizing experimental data, which makes it possible not only to systematize and evaluate experimental data, but also to predict results for little studied materials. We have performed a generalization of experimental data on the thermal conductivity of vapors of the paraffin series hydrocarbons and their alcohols.

For processing the λ values with a single temperature dependence the expression

$$\frac{\lambda}{\lambda_{T_b}} = F\left(\frac{T}{T_b}\right), \tag{1}$$

was used, where T_b is the boiling point of the substance at normal pressure, K [2]; λ_{T_b} is the thermal conductivity of the material at the boiling point, W/(m·K).

The experimental points fit a single curve well in such coordinates. The maximum deviation from the averaging curve for either the paraffins or the alcohols did not exceed ±2%. Aside from the data obtained by the present author in [3], the generalization used data of [4] for hexane, heptane, octane, nonane, and decane, and data from [5, 6] for methyl, ethyl,

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